ACOUSTIC EMISSION FROM SUBSONIC TURBULENT JETS

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A method is proposed for determining the noise intensity of subsonic jets. The results are given from a calculation of the far-field intensity level of a jet for various Mach numbers at the nozzle orifice. The results of the calculation are compared with the experimental data.

The foundations of the theory of turbulent jets have been laid in the work of Lighthill and further elaborated by his followers. According to the theory set forth in [1, 2] the acoustic emission intensity of a jet is proportional to U_a^8. However, the experimental data [3, 4, et al.] indicate that the jet noise intensity for $M_a < 0.6$ is proportional to U_a^6; as M_a is increased the power exponent grows larger, approaching a value of 7 to 7.5 for transonic jets. Several papers have been published to date (e.g., [5, 6]) in which information is presented on turbulence characteristics, permitting a numerical calculation of the intensity level in the far field of a jet according to [1, 2]. The results of this calculation prove to be too low in comparison with the experimental data: by 20 to 25 dB for jets with M_a in the interval from 0.3 to 0.6. The discrepancy of the analytical and experimental results is probably attributable to the fact that fluctuation terms of the type $p'(\partial U_i/\partial x_j)$ were not considered in [2]. Their incorporation into the form of solution adopted in [1, 2] poses difficulties. We propose therefore, to seek an expression for the noise intensity in the field of a jet by a somewhat different method.

We start with the wave equation derived by Lighthill*:

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \Delta \rho = \frac{\partial^2}{\partial x_i \partial x_j} T_{ij}, \tag{1}$$

in which

$$T_{ij} = \rho u_i u_j + p\sigma_{ij} - a_0^2 \rho \sigma_{ij}. \tag{2}$$

Given the initial conditions

$$\rho(\mathbf{r}_0, 0) = \rho_0, \quad \left.\frac{\partial \rho}{\partial t}\right|_{t=0} = 0$$

the solution of (1) can be written in the form

$$\rho(\mathbf{r}_0, t) - \rho_0 = \frac{1}{4\pi a_0^2} \iiint\limits_W \left[\frac{\partial^2}{\partial x_i \partial x_j} T_{ij}\right]_\tau \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}. \tag{3}$$

The integration in (3) is carried out over the volume occupied by the gas existing in turbulent motion, i.e., over the jet volume. The quantity $\tau = t - |\mathbf{r}_0 - \mathbf{r}|/a_0$ is the time at which a signal emitted at the point $\mathbf{r}$ arrives at the point $\mathbf{r}_0$ at time t (Fig. 1). If we neglect the viscosity and sound absorption due to thermal conduction in (2), we find that the expression for the excess pressure takes the form

*Here and elsewhere the double subscript in a single-term expression indicates summation on its values from 1 to 3.

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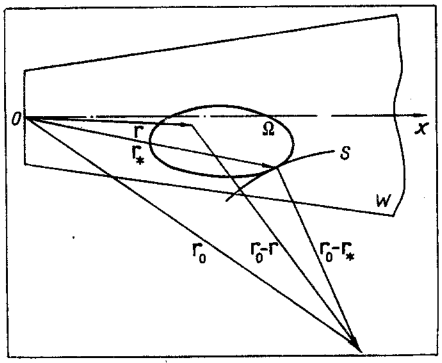

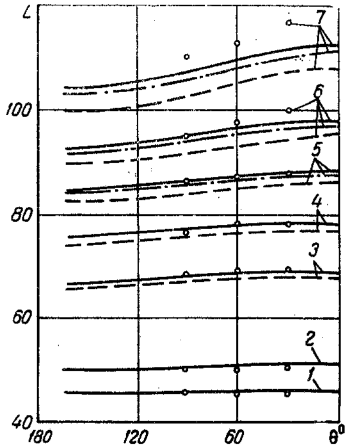

Fig. 1

Fig. 2

Fig. 1. Physical model of the problem.

Fig. 2. Far-field jet noise intensity level versus emission angle and Mach number. 1) M_a = 0 10; 2) 0.12; 3) 0.223; 4) 0.312; 5) 0.435; 6) 0.589; 7) 0.90; dashed curve) first approximation; dot-dash curve) second approximation; solid curve) third approximation; the points denote the experimental data of [3]; L, dB.

$$p(\mathbf{r}_0, t) - p_0 = \frac{1}{4\pi} \iiint\limits_W \left[\frac{\partial^2}{\partial x_i \partial x_j} \rho u_i u_j \right]_\tau \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}.$$

We take the time average, assuming that the average parameters in the jet are time-independent. In this case we obtain for the average excess pressure at $\mathbf{r}_0$, which does not contribute to the noise,

$$P(\mathbf{r}_0) - p_0 = \frac{1}{4\pi} \iiint\limits_W \frac{\partial^2}{\partial x_i \partial x_j} \rho (U_i U_j + \overline{u_i' u_j'}) \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|},$$

where the bar over $u_i'u_j'$ denotes averaging.

For the pressure fluctuation at point $\mathbf{r}_0$ we have

$$p'(\mathbf{r}_0, t) = \frac{1}{4\pi} \iiint\limits_W \left[\frac{\partial^2}{\partial x_i \partial x_j} \rho (U_i u_j' + U_j u_i' + u_i' u_j' - \overline{u_i' u_j'}) \right]_\tau \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}. \tag{4}$$

We consider the gas in the jet to be incompressible and neglect in (4) the difference $u_i'u_j' - \overline{u_i'u_j'}$. Then (4) reduces to the form

$$p'(\mathbf{r}_0, t) = \frac{\rho_0}{2\pi} \iiint\limits_W \frac{\partial U_i}{\partial x_j} \left[\frac{\partial u_j'}{\partial x_i} \right]_\tau \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}.$$

We transform from Cartesian to cylindrical coordinates and in the sum contained in the integrand of the latter equation reject higher-order small terms with regard for the fact that in the jet U ≫ V and ∂U /∂r ≫ ∂U/∂r. Then

$$p'(\mathbf{r}_0, t) = \frac{\rho_0}{2\pi} \iiint\limits_W \frac{\partial U}{\partial r} \left[\frac{\partial v'}{\partial x} \right]_\tau \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}. \tag{5}$$

To calculate the jet noise intensity we use the fundamental principle of Lighthill's theory: if the noise sources in the jet are sufficiently correlated, the pressure amplitude at point $\mathbf{r}_0$ is a linear combination of the amplitudes generated by the sources, and for uncorrelated sources the acoustic energy is additive. In accordance with this principle we partition the jet into domains in which the velocity fluctuations are correlated and we determine the intensity generated at $\mathbf{r}_0$ by a single such domain Ω. To do this we represent τ as follows:

$$\tau(\mathbf{r}) = t - \frac{|\mathbf{r}_0 - \mathbf{r}_*|}{a_0} + \delta t(\mathbf{r}),$$

where $\delta t(\mathbf{r})$ is the propagation time of a perturbation from point $\mathbf{r}$ of the domain Ω to the surface S of a sphere of radius $|\mathbf{r}_0 - \mathbf{r}_*|$ with center at $\mathbf{r}_0$ and tangent to Ω (Fig. 1). We call $\delta t(\mathbf{r})$ the proper time delay by analogy with the term used in electromagnetic field theory [7]. The quantity $|\mathbf{r}_0 - \mathbf{r}_*|/a_0$ represents the propagation time of a perturbation from the surface S to the point $\mathbf{r}_0$.

We expand the function $\partial v'/\partial x$, which depends on the proper delay, in a Taylor series in the neighborhood of the time $t_* = t - |\mathbf{r}_0 - \mathbf{r}|/a_0$:

$$\left[\frac{\partial v'}{\partial x}\right]_\tau = \left[\frac{\partial v'}{\partial x}\right]_{t_*} + \delta t\left[\frac{\partial}{\partial \tau}\left(\frac{\partial v'}{\partial x}\right)\right]_{t_*} + \frac{(\delta t)^2}{2}\left[\frac{\partial^2}{\partial \tau^2}\left(\frac{\partial v'}{\partial x}\right)\right]_{t_*} + \cdots \tag{6}$$

The x derivatives can be expressed in terms of the t derivatives on the basis of the Taylor hypothesis [8]:

$$\frac{\partial}{\partial x} = \frac{1}{U_c} \cdot \frac{\partial}{\partial t}. \tag{7}$$

It now remains for us to consider the relationship between the t and the τ derivatives. Proceeding from the definition of τ, we have

$$\frac{\partial \tau}{\partial t} = 1 - \frac{1}{a_0} \cdot \frac{\partial}{\partial t}|\mathbf{r}_0 - \mathbf{r}|; \quad \frac{\partial}{\partial t}|\mathbf{r}_0 - \mathbf{r}| = -\mathbf{U}_c\mathbf{n}\frac{\partial \tau}{\partial t},$$

where

$$\mathbf{n} = \frac{\mathbf{r}_0 - \mathbf{r}}{|\mathbf{r}_0 - \mathbf{r}|}; \quad \mathbf{U}_c = -\frac{\partial(\mathbf{r}_0 - \mathbf{r})}{\partial t}.$$

We therefore arrive at the relation

$$\frac{\partial}{\partial t} = (1 - M_c \cos\theta)^{-1}\frac{\partial}{\partial \tau}, \tag{8}$$

which describes the Doppler effect, i.e., relates the natural frequency of the sound emitted by a moving source to the frequency perceived in a fixed coordinate system [7].

Taking (6)-(8) into account, we find for the pressure generated by the domain Ω

$$p'(\mathbf{r}_0, t) = \iiint_\Omega B(\mathbf{r})\,[A(\mathbf{r})]_{t_*}\,dW, \tag{9}$$

where

$$B(\mathbf{r}) = \frac{\rho_0}{2\pi} \cdot \frac{\partial U}{\partial r} \cdot \frac{(1 - M_c\cos\theta)^{-1}}{U_c|\mathbf{r}_0 - \mathbf{r}|},$$

$$A(\mathbf{r}) = \frac{\partial v'}{\partial \tau} + \delta t(\mathbf{r})\frac{\partial^2 v'}{\partial \tau^2} + \frac{1}{2}[\delta t(\mathbf{r})]^2\frac{\partial^3 v'}{\partial \tau^3} + \cdots \tag{10}$$

Note that in (9) $A(\mathbf{r})$ is evaluated for all points of Ω at one given time t_*, as opposed to (5), in which the integrand is evaluated at different times for different points of the domain of integration. The choice of the time t_* does not affect the value of the noise intensity, so that in (9) we can drop the argument t_* on the right and t on the left.

We now consider the fluctuation pressure in the zeroth approximation, which is determined by the first term in (10). In this approximation the relative delay of perturbations emitted by different elements of Ω is ignored. Evaluating the integrand in this case by means of the momentum conservation equations for an incompressible gas, we obtain

$$p'(\mathbf{r}_0) = -\frac{1}{4\pi}\iiint_\Omega \Delta p\,\frac{dW}{|\mathbf{r}_0 - \mathbf{r}|}.$$

This integral is equal to zero, because $\mathbf{r}_0$ lies outside the domain Ω [9]. Therefore, the first term of the series (10) does not contribute to the noise pressure.

We further the determination of the jet noise intensity in accordance with [1]. We write Eq. (9) in the approximate form

$$p'(\mathbf{r}_0) = B(\mathbf{r}_c)\,A(\mathbf{r}_c)\,\Omega.$$

Now the intensity of the noise generated by Ω is

$$I_\Omega(\mathbf{r}_0) = \frac{1}{\rho_0 a_0} B^2(\mathbf{r}_c)\,\overline{A(\mathbf{r}_c)\,A(\mathbf{r}_c + \delta\mathbf{r})}\,\Omega^2.$$

The intensity emitted by unit volume of the jet is written in the form

$$I_e(\mathbf{r}_0) = \frac{1}{\rho_0 a_0} B^2(\mathbf{r}_c)\,\overline{A^2(\mathbf{r}_c)}\,\Omega_1, \tag{11}$$

where Ω_1 is the characteristic volume determined by the integral turbulence scales. The intensity of the noise generated at the point $\mathbf{r}_0$ by the entire jet is deduced by integration of (11) over the volume occupied by the turbulent jet:

$$I(\mathbf{r}_0) = \frac{\rho_0}{4\pi^2 a_0} \iiint\limits_W \left(\frac{\partial U}{\partial r}\right)^2 \frac{(1 - M_c \cos\theta)^{-2}}{U_c^2}\,\overline{A^2}\,\Omega_1 \frac{dW}{|\mathbf{r}_0 - \mathbf{r}|^2}, \tag{12}$$

where the expression for $\overline{A^2}$ with regard for the relations

$$\overline{\left(\frac{\partial v'}{\partial \tau}\right)^2} = \omega^2 \overline{v'^2}; \quad \overline{\left(\frac{\partial^2 v'}{\partial \tau^2}\right)^2} = \omega^4 \overline{v'^2} \text{ etc.,}$$

assumes the form

$$\overline{A^2} = \overline{v'^2}\left[(\delta t)^2 \omega^4 + (\delta t)^3 \omega^5 + \frac{7}{12}(\delta t)^4 \omega^6 + \cdots\right]. \tag{13}$$

It is clear that the characteristic time delay δt is proportional to the characteristic dimension of the domain Ω. We assume in the calculations that $\delta t = 0.37 L_x / a_0$.

The calculations were carried out on a digital computer according to Eqs. (12) and (13) for the jet far field, where $|\mathbf{r}_0 - \mathbf{r}|$ was replaced by $|\mathbf{r}_0|$. The requisite parameters were taken from [5, 6, 10]; the necessary turbulence characteristics were specified as approximations of the empirical relations given in the same papers.

The results of the noise intensity level (L) calculations are shown in Fig. 2. As the figure indicates, for jets with $M_a < 0.5$ the calculation of the intensity in the first approximation, i.e., with only the first term retained in (13), yields good agreement with the experimental data, and the subsequent approximations converge rapidly. The noise intensity for the given values of M_a is proportional to U_a^6. For transonic jets the third or fourth approximation suffices, but for better concurrence with the experimental, obviously, the compressibility of the gas in the jet should be taken into account.

NOTATION

t	is the time;
$\mathbf{r}$	is the radius vector;
x_i, x_j	are the projections of the latter on the Cartesian coordinate axes;
$\mathbf{r}_0$	is the radius vector at which the noise intensity is sought;
x, r, φ	are the cylindrical coordinates;
u_i, U_i, u_i'	are the instantaneous, average, and fluctuation velocities in projection on the Cartesian coordinate axes;
u, v, w	are the axial, radial, and tangential components of the instantaneous velocity;
U, V	are the longitudinal and radial average velocities;
v'	is the radial fluctuation velocity component;
a_0	is the velocity of sound;
ρ	is the density;

ρ_0, p_0	are the density and pressure in the unperturbed medium;
p, P, p'	are the instantaneous, average, and fluctuation pressures;
U_a, M_a	are the average velocity and Mach number at the nozzle orifice;
U_c, M_c	are the velocity and Mach number for vortex convection;
Ω	is the characteristic volume of velocity fluctuation correlation domain;
$\mathbf{r}_c$	is the radius vector of the center of the domain Ω;
ω, δt	are the characteristic natural frequency and proper time delay for Ω;
L_x, L_r	are the longitudinal and radial integral turbulence scales;
$\Omega_1 = (\pi/4) L_x L_r^2$;	
$\mathbf{n} = (\mathbf{r}_0 - \mathbf{r})/\lvert \mathbf{r}_0 - \mathbf{r} \rvert$	is the unit vector in the direction of emission;
θ	is the angle between **n** and the x axis;
I	is the noise intensity;
L	is the intensity level;
σ_{ij}	is the unit tensor;
Δ	is the Laplace operator.

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